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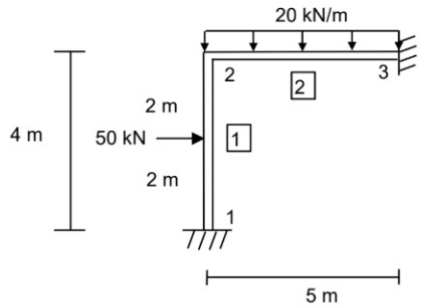
Determine the reactions and member end forces for the frame shown by using the matrix stiffness method.



E = 200 GPa
I = 400x10^6 mm^4
A = 7500 mm^2

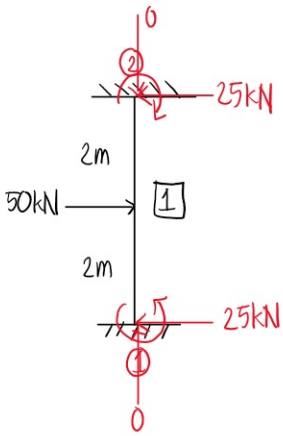
$E = 200GPa\,(1000^2) = 200\times10^6\ kPa$

$I = 400\times10^6\ mm^4\left(\frac{1}{1000^4}\right) = 0.0004\ m^4$

$A = 7500\left(\frac{1}{1000^2}\right) = 0.0075\ m^2$

$L_1 = 4m$

$L_2 = 5m$



$Qf_1 = -\frac{50}{2} = -25 = Qf_4$

$Qf_2 = Qf_5 = 0$

$Qf_3 = Qf_6 = \frac{50(2)(2)^2}{4^2} = 25\ kN\cdot m$

$$F_{f_1} = \begin{bmatrix} -25 \\ 0 \\ 25 \\ -25 \\ 0 \\ -25 \end{bmatrix} \begin{matrix} 0 \\ 0 \\ 0 \\ 1 \\ 2 \\ 3 \end{matrix}$$

$$k_1 = \frac{EI}{L_1^3}\begin{bmatrix} \frac{AL^2}{I} & 0 & 0 & -\frac{AL^2}{I} & 0 & 0 \\ 0 & 12 & 6L & 0 & -12 & 6L \\ 0 & 6L & 4L^2 & 0 & -6L & 2L^2 \\ -\frac{AL^2}{I} & 0 & 0 & \frac{AL^2}{I} & 0 & 0 \\ 0 & -12 & -6L & 0 & 12 & -6L \\ 0 & 6L & 2L^2 & 0 & -6L & 4L^2 \end{bmatrix} = \begin{matrix} 0 & 0 & 0 & 1 & 2 & 3 \end{matrix}\begin{bmatrix} 37500 & 0 & 0 & -37500 & 0 & 0 \\ 0 & 15000 & 30000 & 0 & -15000 & 30000 \\ 0 & 30000 & 80000 & 0 & -30000 & 40000 \\ -37500 & 0 & 0 & 37500 & 0 & 0 \\ 0 & -15000 & -30000 & 0 & 15000 & -30000 \\ 0 & 30000 & 40000 & 0 & -30000 & 80000 \end{bmatrix} \begin{matrix} 0 \\ 0 \\ 0 \\ 1 \\ 2 \\ 3 \end{matrix}$$

$\cos\theta = 0$

$\sin\theta = 1$ (vertical member)

$$T = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T^T = \begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

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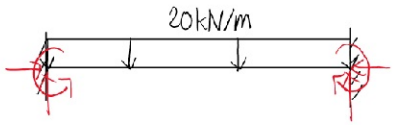
$$k_1 = T^T k \cdot T = \begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 375000 & 0 & 0 & -375000 & 0 & 0 \\ 0 & 15000 & 30000 & 0 & -15000 & 30000 \\ 0 & 30000 & 80000 & 0 & -30000 & 40000 \\ -375000 & 0 & 0 & 375000 & 0 & 0 \\ 0 & -15000 & -30000 & 0 & 15000 & -30000 \\ 0 & 30000 & 40000 & 0 & -30000 & 80000 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{array}{c} \begin{matrix} 0 & 0 & 0 & 1 & 2 & 3 \end{matrix} \\ \begin{bmatrix} 15000 & 0 & -30000 & -15000 & 0 & -30000 \\ 0 & 375000 & 0 & 0 & -375000 & 0 \\ -30000 & 0 & 80000 & 30000 & 0 & 40000 \\ -15000 & 0 & 30000 & 15000 & 0 & 30000 \\ 0 & -375000 & 0 & 0 & 375000 & 0 \\ -30000 & 0 & 40000 & 30000 & 0 & 80000 \end{bmatrix} \end{array} \begin{matrix} 0 \\ 0 \\ 0 \\ 1 \\ 2 \\ 3 \end{matrix}$$



$$Qf_1 = 0$$

$$Qf_2 = \frac{20(5)}{2} = 50$$

$$Qf_3 = \frac{20(5)^2}{12} = 41.67$$

$$Qf_4 = 0$$

$$Qf_5 = 50$$

$$Qf_6 = -Qf_3 = -41.67$$

$$Ff_2 = \begin{bmatrix} 0 \\ 50 \\ 41.67 \\ 0 \\ 50 \\ -41.67 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 0 \\ 0 \\ 0 \end{matrix}$$

$$k_2 = \frac{EI}{L_2^3} \begin{bmatrix} \frac{AL^2}{I} & 0 & 0 & -\frac{AL^2}{I} & 0 & 0 \\ 0 & 12 & 6L & 0 & -12 & 6L \\ 0 & 6L & 4L^2 & 0 & -6L & 2L^2 \\ -\frac{AL^2}{I} & 0 & 0 & \frac{AL^2}{I} & 0 & 0 \\ 0 & -12 & -6L & 0 & 12 & -6L \\ 0 & 6L & 2L^2 & 0 & -6L & 4L^2 \end{bmatrix} = \begin{array}{c} \begin{matrix} 1 & 2 & 3 & 0 & 0 & 0 \end{matrix} \\ \begin{bmatrix} 300000 & 0 & 0 & -300000 & 0 & 0 \\ 0 & 7680 & 19200 & 0 & -7680 & 19200 \\ 0 & 19200 & 64000 & 0 & -19200 & 32000 \\ -300000 & 0 & 0 & 300000 & 0 & 0 \\ 0 & -7680 & -19200 & 0 & 7680 & -19200 \\ 0 & 19200 & 32000 & 0 & -19200 & 64000 \end{bmatrix} \end{array} \begin{matrix} 1 \\ 2 \\ 3 \\ 0 \\ 0 \\ 0 \end{matrix}$$

$$Ff_1 = 0$$

Joint Load Vector:

$$P_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \end{matrix}$$

Fixed-Joint Force Vector:

$$P_f = Ff_1 + Ff_2$$

$$P_f = \begin{bmatrix} -25+0 \\ 0+50 \\ -25+41.67 \end{bmatrix} = \begin{bmatrix} -25 \\ 50 \\ 16.67 \end{bmatrix}$$

$$S = \begin{bmatrix} 15000+300000 & 0+0 & 30000+0 \\ 0+0 & 375000+7680 & 0+19200 \\ 30000+0 & 0+19200 & 80000+64000 \end{bmatrix} = \begin{bmatrix} 315000 & 0 & 30000 \\ 0 & 382680 & 19200 \\ 30000 & 19200 & 144000 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \end{matrix}$$

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$$P - P_f = Sd$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} -25 \\ 50 \\ 16.67 \end{bmatrix} = \begin{bmatrix} 315000 & 0 & 30000 \\ 0 & 382680 & 19200 \\ 30000 & 19200 & 144000 \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

$$\boxed{\begin{aligned} d_1 &= 0.0000906\ mm \\ d_2 &= -0.000125\ mm \\ d_3 &= -0.000118\ rad \end{aligned}}$$ //

$$u_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.0000906 \\ -0.000125 \\ -0.000118 \end{bmatrix} \qquad u_2 = \begin{bmatrix} 0.0000906 \\ -0.000125 \\ -0.000118 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$Q = Ku + Qf$$

$$Q_1 = \begin{bmatrix} 15000 & 0 & -30000 & -15000 & 0 & -30000 \\ 0 & 375000 & 0 & 0 & -375000 & 0 \\ -30000 & 0 & 80000 & 30000 & 0 & 40000 \\ -15000 & 0 & 30000 & 15000 & 0 & 30000 \\ 0 & -375000 & 0 & 0 & 375000 & 0 \\ -30000 & 0 & 40000 & 30000 & 0 & 80000 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.0000906 \\ -0.000125 \\ -0.000118 \end{bmatrix} + \begin{bmatrix} -25 \\ 0 \\ 25 \\ -25 \\ 0 \\ -25 \end{bmatrix} = \begin{bmatrix} -22.82 \\ 46.78 \\ 23 \\ -27.18 \\ -46.78 \\ -31.72 \end{bmatrix}$$

$$Q_2 = \begin{bmatrix} 300000 & 0 & 0 & -300000 & 0 & 0 \\ 0 & 7680 & 19200 & 0 & -7680 & 19200 \\ 0 & 19200 & 64000 & 0 & -19200 & 32000 \\ -300000 & 0 & 0 & 300000 & 0 & 0 \\ 0 & -7680 & -19200 & 0 & 7680 & -19200 \\ 0 & 19200 & 32000 & 0 & -19200 & 64000 \end{bmatrix} \begin{bmatrix} 0.0000906 \\ -0.000125 \\ -0.000118 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 50 \\ 41.67 \\ 0 \\ 50 \\ -41.67 \end{bmatrix} = \begin{bmatrix} 27.18 \\ 46.78 \\ 31.72 \\ -27.18 \\ 53.22 \\ -47.84 \end{bmatrix}$$

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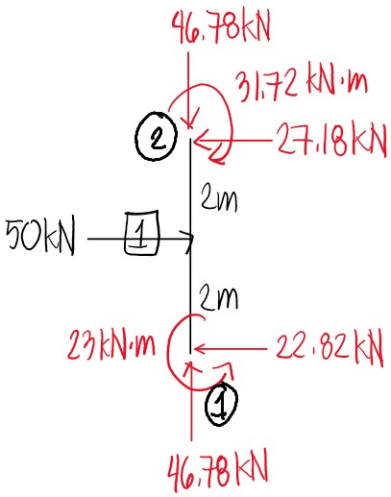

Original Beam